

Entanglement dynamics of a qubit-qutrit system in non-inertial frames

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The effect of decoherence on a qubit-qutrit system under the influence of global and multilocal decoherence in non-inertial frames is investigated. It is shown that the entanglement sudden death (ESD) can be avoided in non-inertial frames in the presence of dephasing and bit-trit phase flip channels for entire range of decoherence. However, ESD behaviour is seen for higher level of decoherence in case of phase flip and bit-trit flip channels. Irrespective of the entanglement degradation caused by the Unruh effect, no ESD occurs for the dephasing environment. Furthermore, flipping channels have approximately similar effect on the entanglement of the hybrid system.

Keywords: Quantum decoherence; entanglement dynamics; non-inertial frames.

I. INTRODUCTION

Quantum entanglement plays an important role in various quantum information tasks, such as quantum teleportation [1] and quantum computation [2]. However entangled states are always coupled with the environment due to the unavoidable decoherence. As a result of the system-environment interaction, the pure entangled states become mixed states and therefore become quite useless for quantum information tasks [3]. Multi-partite entangled states of qubits and qutrits are a central resource of quantum information science. They can be used in constructing different protocols, for example, key distribution and quantum computation [4, 5]. During recent years, two important phenomenon, entanglement sudden death (ESD) and entanglement sudden birth (ESB) have been investigated for bipartite and multipartite states [6-13]. In this context, Yu and Eberly [14, 15] have investigated the time evolution of entanglement of a bipartite qubit system undergoing various modes of decoherence. Peres-Horodecki [16, 17] have studied entanglement

of qubit-qubit and qubit-qutrit states and established separability criterion. According to this criterion, the partial transpose of a separable density matrix must have non-negative eigenvalues, where the partial transpose is taken over the smaller subsystem for qubit-qutrit case. Ann et al. [18] have studied the ESD behaviour of a qubit-qutrit system under the influence of dephasing noise.

Relativistic quantum information combines the tools from general relativity, quantum field theory and quantum information theory. It is a rather new and fast-growing field. It has attracted much attention during recent years to study Unruh or Hawking effect on the entanglement shared between inertial and non-inertial observers [19–30]. However, most of the investigations in non-inertial frames are related to the study of the quantum information in an isolated system. The decoherence [31], which appears when a system interacts with its environment in an irreversible way, can be viewed as the transfer of information from the system to its environment. It plays a fundamental role in the description of the quantum-to-classical transition [32] and has been successfully applied in the cavity QED [33] and ion trap experiments [34]. Recently, implementation of decoherence in non-inertial frames have been investigated for bipartite and tripartite systems [35, 36].

In this paper, the effect of decoherence on a qubit-qutrit system in non-inertial frames by considering different noise models, such as, phase flip, dephasing, bit-trit flip and bit-trit phase flip channels, parameterized by decoherence parameter p such that $p \in [0, 1]$. The lower and upper limits of the decoherence parameter correspond to the fully coherent and fully decohered system, respectively. Different couplings of the system and the environment where the system is influenced by global and multi-local noises are considered. It is shown that the ESD can be avoided in non-inertial frames in the presence of dephasing and bit-trit phase flip environments. However, no ESD occurs for the dephasing environment.

II. QUBIT-QUTRIT SYSTEM IN NON-INERTIAL FRAMES

Let a composite system of qubit A and qutrit B is coupled to a noisy environment both collectively and globally. Multi-local coupling describes the situation when the qubit and qutrit are independently influenced by their individual noisy environments. Whereas, the global decoherence corresponds to the situation when it is influenced by both collective and multilocal noises at the same time. The term collective coupling means when both the qubit and qutrit are influenced by the same noise. In this study, a particular initial state of the qubit-qutrit system is considered.

The density matrix can be computed by taking into account that Rob is constrained to region I of Rindler space-time, that requires to rewrite Rob's mode in terms of Rindler modes and then to trace over the unobservable Rindler's region IV . Therefore, the state shared between the two parties is an Unruh-Rindler hybrid (qubit-qutrit system) entangled state as given by [37]

$$\rho_{AR} = \frac{1}{2} \begin{pmatrix} \cos^2 r (|01\rangle_{AR} \langle 01| + |01\rangle_{AR} \langle 10| + |10\rangle_{AR} \langle 01| + |10\rangle_{AR} \langle 10|) \\ + \sin^2 r (|02\rangle_{AR} \langle 02| + |12\rangle_{AR} \langle 12|) \end{pmatrix}. \quad (1)$$

Alice has a qubit and Rob would have a qutrit, since for Rob's mode he could have three different possible orthogonal states: particle spin-up, particle spin-down and particle pair and the Minkowski-Rindler modes, the subscripts A and R correspond to Alice and Rob respectively. The notation 0, 1, and 2 correspond to spin up \uparrow , spin down \downarrow and spin-up/down $\uparrow\downarrow$ states respectively.

Let us assume that Alice remain stationary while Rob moves with uniform acceleration. It is important to note that Rob, when he is accelerated with respect to an inertial observer of the Dirac vacuum would observe a thermal distribution of fermionic spin 1/2 particles when he observes the Minkowski vacuum due to Unruh effect [37]. The above state is obtained after taking the trace over unobserved region IV [38]. The evolution of a state of a quantum system in a noisy environment can be described by the super-operator in the Kraus operator representation as [39]

$$\rho_f = \sum_k E_k \rho_i E_k^\dagger, \quad (2)$$

where the Kraus operators E_i satisfy the following completeness relation

$$\sum_k E_k^\dagger E_k = I. \quad (3)$$

The single qubit Kraus operators for phase flip, dephasing, bit flip and bit phase flip channels are respectively given in table 1. Whereas the single qutrit Kraus operators for the phase flip channel are given by

$$E_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-p} & 0 \\ 0 & 0 & \sqrt{1-p} \end{pmatrix}, \quad E_1 = \begin{pmatrix} 0 & \sqrt{p} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 0 & 0 & \sqrt{p} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (4)$$

and the single qutrit Kraus operators for dephasing channel are given as

$$E_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-p} & 0 \\ 0 & 0 & \sqrt{1-p} \end{pmatrix}, \quad E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{p} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sqrt{p} \end{pmatrix} \quad (5)$$

Whereas the single qutrit Kraus operators for trit flip channel are given by

$$E_0 = \sqrt{1 - \frac{2p}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad E_1 = \sqrt{\frac{p}{3}} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad E_2 = \sqrt{\frac{p}{3}} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad (6)$$

and the single qutrit Kraus operators for the trit phase flip channel are given by

$$E_0 = \sqrt{1 - \frac{2p}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad E_1 = \sqrt{\frac{p}{3}} \begin{pmatrix} 0 & 0 & e^{\frac{2\pi i}{3}} \\ 1 & 0 & 0 \\ 0 & e^{-\frac{2\pi i}{3}} & 0 \end{pmatrix},$$

$$E_2 = \sqrt{\frac{p}{3}} \begin{pmatrix} 0 & e^{-\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{2\pi i}{3}} \\ 1 & 0 & 0 \end{pmatrix}, \quad E_3 = \sqrt{\frac{p}{3}} \begin{pmatrix} 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{-\frac{2\pi i}{3}} \\ 1 & 0 & 0 \end{pmatrix} \quad (7)$$

The evolution of the initial density matrix of the composite system when it is influenced by local and multi-local environments is given in Kraus operator form as

$$\rho_f = \sum_{i,j,k} (E_j^B E_k^A) \rho_{AR} (E_j^B E_k^A)^\dagger \quad (8)$$

and the evolution of the system when it is influenced by global environment is given in Kraus operator representation as

$$\rho_f = \sum_{i,j,k} (E_i^{AB} E_j^B E_k^A) \rho_{AR} (E_i^{AB} E_j^B E_k^A)^\dagger \quad (9)$$

where $E_k^A = E_m^A \otimes I_3$, $I_2 \otimes E_j^B$ are the Kraus operators of the multilocal coupling of qubit and qutrit individually and $E_i^{AB} = E_m^A \otimes E_n^A$ are the Kraus operators of the collective coupling of the qutrit system. Using equations (4-9) along with the initial density matrix of as given in equation (1) and taking the partial transpose over the smaller subsystem (qubit), the eigenvalues of the final density matrix can be easily found. Let the decoherence parameters for local and global noise of

the qubit and qutrit be p_1 , p_2 and p respectively. The entanglement for all mixed states ρ_{AB} of a qubit-qutrit system is well quantified by the negativity [40]

$$N(\rho_{AB}) = \max\{0, \sum_k |\lambda_k^{TA(-)}|\} \quad (10)$$

where $\lambda_k^{TA(-)}$ represents the negative eigenvalues of the partial transpose of the density matrix ρ_{AB} with respect to the smaller subsystem.

III. RESULTS AND DISCUSSIONS

The only possible negative eigenvalues of the partial transpose matrix when the system is influenced by the multi-local and global noises of the phase flip channel are given by

$$\begin{aligned} \lambda_{ml} &= -\frac{1}{2}\sqrt{(p_1 - 1)^2(p_2 - 1)^2 \cos^4(r)} \\ \lambda_g &= \left(-\frac{1}{2}\sqrt{(p - 1)^2(p_1 - 1)^4(p_2 - 1)^2 \cos^4(r)} \right) \end{aligned} \quad (11)$$

where the subscripts ml and g represent the multi-local and global noise respectively. The negativity can be calculated using equation (10) for all the cases under consideration. The only possible negative eigenvalues of the partial transpose matrix when the system is influenced by the multi-local and global noises of the dephasing channel are given by

$$\begin{aligned} \lambda_{ml} &= -\frac{1}{2}\sqrt{(1 - p_1) \left(2\sqrt{(1 - p_2)p_2} + 1 \right) \cos^4(r)} \\ \lambda_g &= -\frac{1}{2}\sqrt{\left(2\sqrt{(1 - p)p} + 1 \right) (p_1 - 1)^2 \left(2\sqrt{(1 - p_2)p_2} + 1 \right) \cos^4(r)} \end{aligned} \quad (12)$$

The relations for other channels are too lengthy and instead of presenting in the text, I have potted them for analysis. In order to investigate the effect of decoherence on the qubit-qutrit system in non-inertial frames, the negativity is plotted as a function of decoherence parameter, p in figure 1 (a) for Rob's acceleration $r = \pi/6$ (b) $r = \pi/4$ and as a function of Rob's acceleration, r (c) for $p = 0.3$ and (d) $p = 0.7$ for multi-local noise. Here the decoherence parameter, p corresponds to $p_1 = p_2 = p$. It is seen that for multi-local coupling, no ESD is seen even at $r = \pi/4$ for $p < 1$. In figure2, negativity is plotted as a function of decoherence parameter, p in figure 2 (a) for Rob's acceleration $r = \pi/6$ (b) $r = \pi/4$ and as a function of Rob's acceleration, r (c) for $p = 0.3$ and (d) $p = 0.7$ for global noise of different channels. It is seen that in case of global noise, ESD behaviour is seen for $p > 0.6$ for phase flip and bit-trit flip channels. It is also seen that maximal entanglement degradation occurs under global noise. It is also seen that the entanglement is degraded heavily as

we increase the value of Rob's acceleration from $r = \pi/10$ to $r = \pi/4$ (infinite acceleration limit). Furthermore, a similar behaviour of flipping channels is seen towards entanglement degradation.

In figure 3, three dimensional graphs for negativity are plotted as a function of Rob's acceleration, r and decoherence parameter, p influenced by global noise of all the channels under consideration. It is seen that for higher values of decoherence parameters, entanglement sudden death occurs in case of flipping channels for $p > 0.5$. Further more, it is seen that no ESD occurs for any acceleration of Rob for the entire range of decoherence parameters in case of dephasing channel.

IV. CONCLUSIONS

The effect of decoherence on a qubit-qutrit system under the influence of global and multilocal decoherence in non-inertial frames is investigated by considering phase flip, dephasing, bit-trit flip and bit-trit phase flip channels. It is shown that the entanglement sudden death can be avoided in non-inertial frames in the presence of dephasing and bit-trit phase flip channels for entire range of decoherence. ESD behaviour is seen for higher level of decoherence in case of phase flip and bit-trit flip channels. However, no ESD occurs for the dephasing environment. Furthermore, flipping channels have a similar effect on the entanglement of the hybrid system.

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Figures captions

Figure 1. (Color online). The negativity is plotted as a function of decoherence parameter, p in figure 1 (a) for Rob's acceleration $r = \pi/6$ (b) $r = \pi/4$ and as a function of Rob's acceleration, r (c) for $p = 0.3$ and (d) $p = 0.7$ for multi-local noise.

Figure 2. (Color online). The negativity is plotted as a function of decoherence parameter, p in figure 2 (a) for Rob's acceleration $r = \pi/6$ (b) $r = \pi/4$ and as a function of Rob's acceleration, r (c) for $p = 0.3$ and (d) $p = 0.7$ for global noise of different channels.

Figure 3. (Color online). The negativity are plotted as a function of Rob's acceleration, r and decoherence parameter, p influenced by global noise of all the channels under consideration.

Table Caption

Table 1. Single qubit Kraus operators for phase flip, dephasing, bit-trit flip and bit-trit phase flip channels where p represents the decoherence parameter.

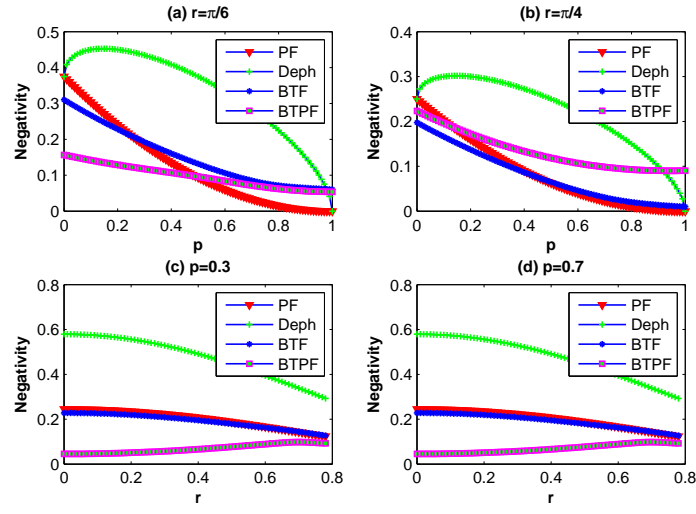


FIG. 1: (Color online) The negativity is plotted as a function of decoherence parameter, p in figure 1 (a) for Rob's acceleration $r = \pi/6$ and (b) $r = \pi/4$ and as a function of Rob's acceleration, r (c) for $p = 0.3$ and (d) $p = 0.7$ for multi-lo

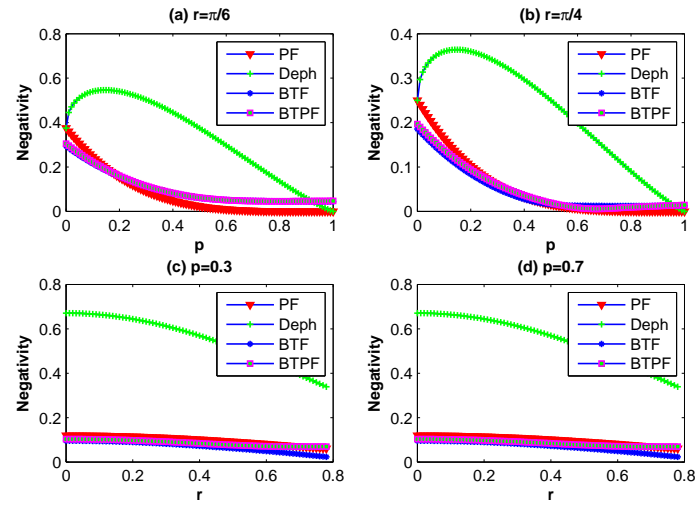


FIG. 2: (Color online). The negativity is plotted as a function of decoherence parameter, p in figure 2 (a) for Rob's acceleration $r = \pi/6$ (b) $r = \pi/4$ and as a function of Rob's acceleration, r (c) for $p = 0.3$ and (d) $p = 0.7$ for global noise of different channels.

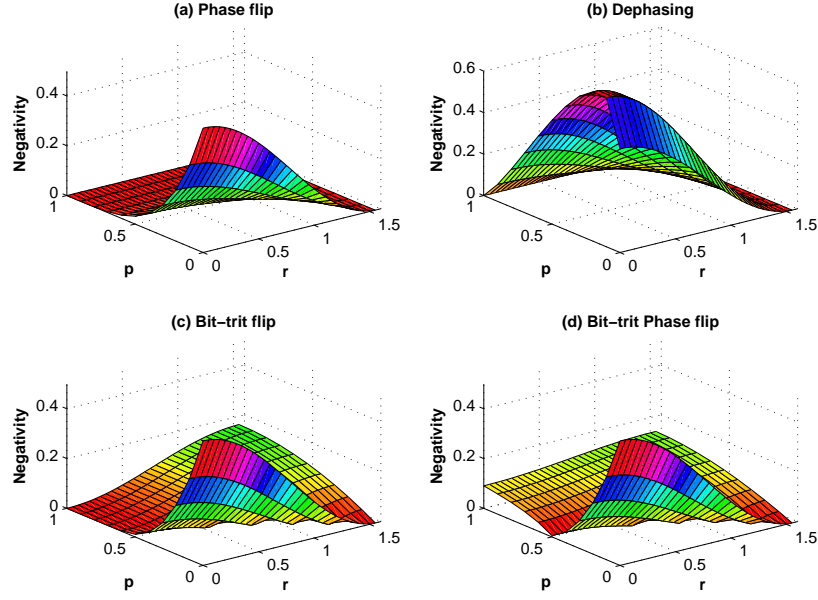


FIG. 3: (Color online). The negativity are plotted as a function of Rob's acceleration, r and decoherence parameter, p influenced by global noise of all the channels under consideration.

TABLE I: Single qubit Kraus operators for phase flip, dephasing, bit-trit flip and bit-trit phase flip channels where p represents the decoherence parameter.

Phase flip channel	$E_0 = \sqrt{1 - \frac{p}{2}}I, \quad E_1 = \sqrt{\frac{p}{2}} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Dephasing channel	$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0 & \\ 0 & \sqrt{p} \end{bmatrix}$
Bit flip channel	$E_0 = \sqrt{1 - \frac{p}{2}}I, \quad E_1 = \sqrt{\frac{p}{2}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Bit-phase flip channel	$E_0 = \sqrt{1 - \frac{p}{2}}I, \quad E_1 = \sqrt{\frac{p}{2}} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$